2.1 Completed Notes

2.1: Base 10 and Base 5 Numeration Systems

Definition: If a is any number and n is any natural number, then

$a^n = a \times a \times a \times \ldots \times a$ (n factors)

Our number system is called the Hindu-Arabic numeration system, and it is a base 10 number system using the characters 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note that there are 10 characters.)

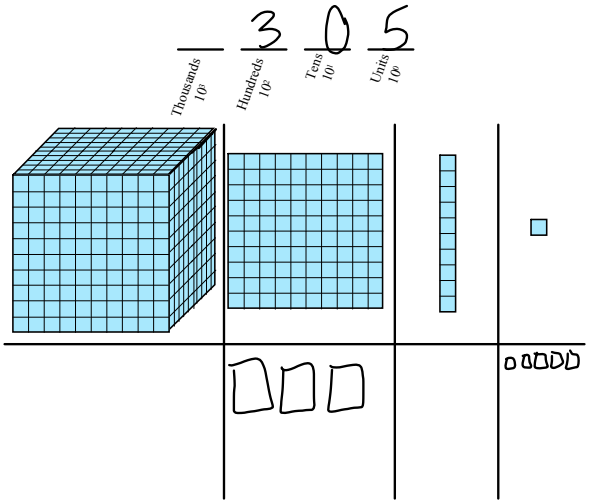
What does this mean? When a number is written in base 10, each "place value" corresponds to a power of 10.

Example: The number 6143 means "6 thousands, 1 hundred, 4 tens, and 3 ones".

6	1	4	3
Thousands 10^3	Hundreds 10^2	Tens 10^1	Units 10^0

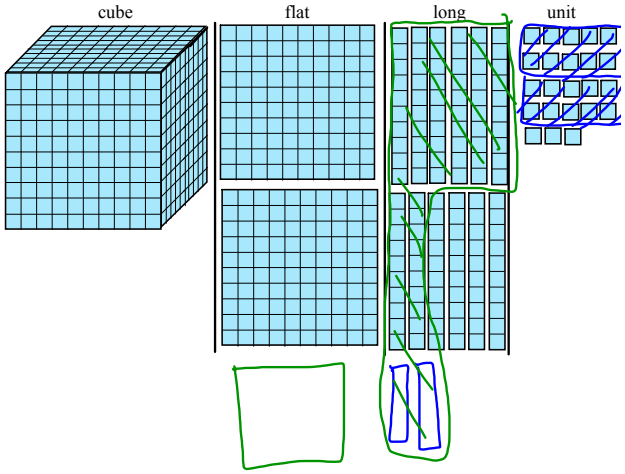
Another perspective: We can also write the number 6143 in expanded form as $6143 = 6 \cdot 10^3 + 1 \cdot 10^2 + 4 \cdot 10^1 + 3 \cdot 10^0$

Example: Represent the number three hundred five in base 10.



Example: If you have 1 cube, 2 flats, 12 longs, and 23 units, what is the minimum number of blocks you can have using a fair trade?

10 units = 1 long 10 longs = 1 flat 10 flats = 1 cube



Example: If you have 1 cube, 2 flats, 12 longs, and 23 units, what is the minimum number of blocks you can have using a fair trade?

Consider filling the diagram below in the same manner. Is this number valid?

1	2	12	23
Thousands 10^3	Hundreds 10^2	Tens 10^1	Units 10^0

We showed that this number is the same as this one:

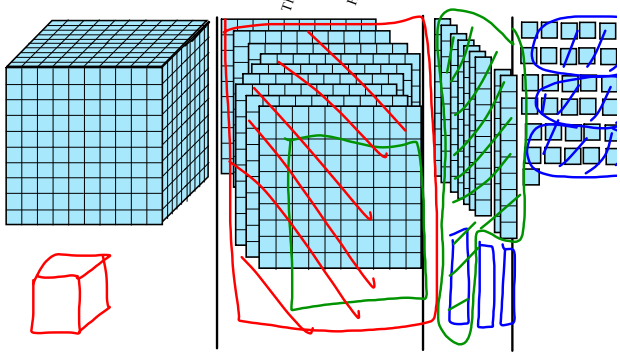
1	3	4	3
Thousands 10^3	Hundreds 10^2	Tens 10^1	Units 10^0

This gives us an important fact about the base 10 number system. You cannot have more than 9 in a single "place value".

Example: If you have 9 flats, 9 longs, and 31 units representing a base 10 number, perform the necessary exchanges to write it in the proper form.

10 units = 1 long
10 longs = 1 flat
10 flats = 1 cube

2	0	2	1
Thousands 10^3	Hundreds 10^2	Tens 10^1	Units 10^0



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Definition: The base 5 number system uses the characters 0, 1, 2, 3, and 4 and each "place value" corresponds to a power of 5.

Notation: We denote a number in base five by writing "five" (preferred) or "5" in a subscript.

Example: The number 2143_{five} means "2 5^3's, 1 5^2, 4 5^1's, and 3 ones".

5^3	5^2	5^1	Units 5^0
2	1	4	3

Let's count the first 30 base 5 numbers:

$1_5, 2_5, 3_5, 4_5, 10_5, 11_5, 12_5, 13_5, 14_5, 20_5, 21_5, 22_5$
$23_5, 24_5, 30_5, 31_5, 32_5, 33_5, 34_5, 40_5, 41_5, 42_5, 43_5$
$44_5, 100_5, 101_5, 102_5, 103_5, 104_5, 110_5$

Example: The number 2143_{five} means "2 5^3's, 1 5^2, 4 5^1's, and 3 ones".

5^3	5^2	5^1	Units 5^0
2	1	4	3

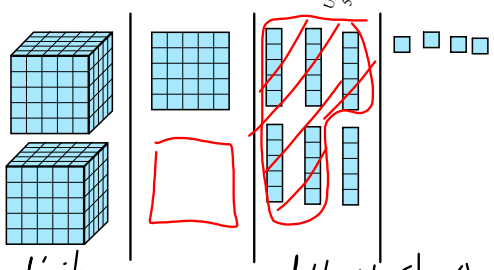
What does this number mean in base 10? Let's try expanded form.

$2 \cdot 5^3 + 1 \cdot 5^2 + 4 \cdot 5^1 + 3 \cdot 5^0$
$2(125) + 25 + 20 + 3$
$= 250 + 25 + 20 + 3 = 298_{ten}$

Note: A number without a base written is assumed to be base ten.

Example: What is wrong with this picture?
General Rule:

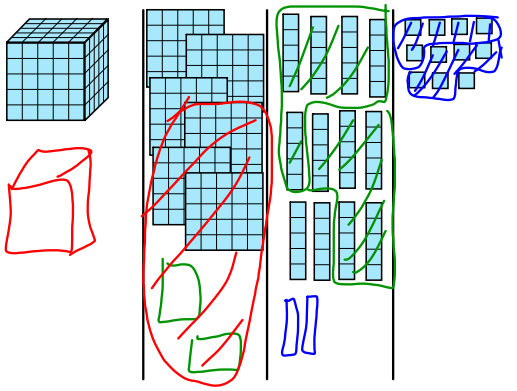
5^3	5^2	5^1	Units 5^0
2	2	1	4 five
2	1	6	4



No digit may ever exceed 4. No 5's, 6's, 7's, 8's, or 9's.

Example: If you have 1 cube, 6 flats, 12 longs, and 11 units, what is the minimum number of blocks you can have using a fair trade?

cube	flat	long	unit five
2	3	4	1



Example: If you have 1 cube, 6 flats, 12 longs, and 11 units, what is the minimum number of blocks you can have using a fair trade?

We showed that this description gives us the following base 5 number:

5^3	5^2	5^1	Units 5^0
2	3	4	1

What is this number in base 10?

$2 \cdot 5^3 + 3 \cdot 5^2 + 4 \cdot 5 + 1$
$2(125) + 3(25) + 20 + 1$
$250 + 75 + 20 + 1 = 346_{ten}$

Conversions: One method to convert a number from base 10 to base 5 uses a form of repeated long division.

Example: Convert 423_{ten} to base 5.

$125 \overline{)\,423\,} \; 3$
-375
$25 \overline{)\,48\,} \; 1$
-25
$5 \overline{)\,23\,} \; 4$
-20
3

5^3	5^2	5^1	Units 5^0
3	1	4	3 five

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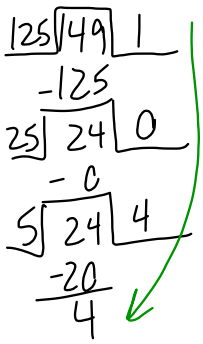
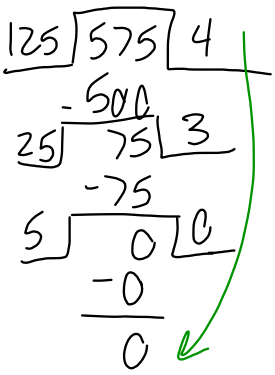
Example: Convert 149_{ten} to base 5.



1044_{five}

Example: Convert 575_{ten} to base 5.

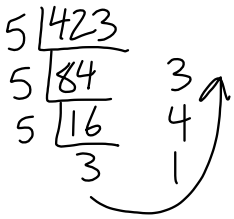


4300_{five}

Example: Convert 423_{ten} to base 5. (This was the first example.)

3143_{five}

Different Method:



3143_{five}

Bonus for a free quiz:

Write up an explanation for why this works and turn it in tomorrow. If someone explains why it works to the class, all of you may use it.